EXERCISES TO THE FOUR EXPOSITIONS ON THE "ALGORITHMIC THEORY OF NUMBERS"

1. Exercises using Paper and Pencil

[CG1] Determine all *reduced positive definite* forms $f = (a, b, c)$ with discriminant
(a) $d = -3$, (b) $d = -4$, (c) $d = -8$, (d) $d = -15$, (e) $d = -20$, (f) $d = -23$.

[CG2] For which *negative fundamental* discriminants $d < 0$ is the *unit form*,

$$f = \begin{cases} (1, 0, -\frac{d}{4}) & \text{if } d \equiv 0 \mod 4, \\ (1, 1, \frac{1-d}{4}) & \text{if } d \equiv 1 \mod 4, \end{cases}$$

a reduced positive definite form ?

[CG3] Determine all *reduced indefinite* forms $f = (a, b, c)$ with discriminant
(a) $d = 5$, (b) $d = 8$, (c) $d = 12$, (d) $d = 13$, (e) $d = 40$.

[CG4] For which *positive fundamental* discriminants $d > 0$ is the unit form f a reduced indefinite form ?

[CG5] Use the chain of *distinguished right neighbors* $(\nu^j(f))_{j\geq 0}$ to reduce the unit form f for the discriminants
(a) $d = 8$, (b) $d = 12$, (c) $d = 13$, (d) $d = 17$, (e) $d = 21$, (f) $d = 32\,009$.

[FU1] Determine the *fundamental unit* $\varepsilon > 1$ and its norm $N_{K|\mathbb{Q}}(\varepsilon)$ for the real quadratic field $K = \mathbb{Q}(\sqrt{d})$ with discriminant
(a) $d = 5$, (b) $d = 8$, (c) $d = 12$, (d) $d = 13$, (e) $d = 57$.

2. Exercises using PARI/GP

[PF1] Compute the following initial sections of *aliquot sequences*.
(a) $(s^i(28))_{0\leq i\leq 4}$,
(b) $(s^i(220))_{0\leq i\leq 5}$,
(c) $(s^i(12\,496))_{0\leq i\leq 8}$,
(d) $(s^i(30))_{0\leq i\leq 15}$,
(e) $(s^i(276))_{0\leq i\leq 53}$.

[PF2] *Factorize* the integer 13 290 059 by means of the CFRAC algorithm.

3. Exercises using MAGMA

[NF1] Compute the abelian type invariants of the 3-class groups of all non-Galois *totally real cubic* fields L with discriminant $0 < d_L < 7 \cdot 10^7$ and of their normal closures N, subject to the constraints

- the primes 3 and 7 split in the unique quadratic subfield K of N,
- the *conductor* f of the cyclic cubic extension $N|K$ equals $3^2 \cdot 7 = 63$.

Observe that the absolute Galois group of N is the *dihedral* group $\mathrm{Gal}(N|\mathbb{Q}) \simeq D_3$ of order 6. That is a non-abelian group.

[NF2] Determine the 3-*capitulation type* (including information on *Taussky*'s conditions A and B) of all complex quadratic fields $K = \mathbb{Q}(\sqrt{d})$ with discriminant $-10^5 < d < 0$ and 3-class group $\mathrm{Cl}_3(K)$ of type $(3, 3)$ in their *unramified* cyclic cubic relative extensions.

4. Exercises using unspecified Information Technology

[CG6] Find *generating forms* for the 3-class group $\mathrm{Cl}_3(K)$ of the complex quadratic fields $K = \mathbb{Q}(\sqrt{d})$ with discriminants $d = -28\,031$ and $d = -4\,447\,704$.

[CG7] Compute all cycles of reduced indefinite forms $f = (a, b, c)$ with discriminant $d = 32\,009$ and determine the *number of disjoint cycles* and the *length of each cycle*.

[FU2] Determine the relative frequency (percentage) of the occurrence of *regulator quotients* $\frac{R_2}{R_1} = 1$ among all real quadratic fields $K = \mathbb{Q}(\sqrt{d})$ with discriminant $0 < d < 10^5$ such that $d \equiv 5 \mod 8$.

Which other values are possible for $\frac{R_2}{R_1}$?