Pattern recognition via Artin transfers, applied to p-class field towers

Daniel C. Mayer[1]

[1]Laboratoire de Mathématique, Université Charles François, Graz, Austria

Résumé/Abstract

The *strategy of pattern recognition* by means of kernels and targets of Artin transfers was founded by myself in 2009 and developed systematically in the past ten years. It is a progressive technique for determining the structure of the various stages, $\mathrm{Gal}(F^{(n)}/F)$, $n \geq 1$, of the p-class tower, $F = F^{(0)} \leq F^{(1)} \leq F^{(2)} \leq \ldots \leq F^{(n)} \leq \ldots$, of an algebraic number field $F/\mathbb{Q}$ for a prime number p. Whereas for $n \geq 3$ non-abelian *iterated Artin patterns of higher order* with increasing complexity are required, it suffices to know the abelian *Artin pattern of first order*, $\mathrm{AP}(G) = (\kappa(G), \tau(G))$, for the identification of the metabelianization, that is the second derived quotient, $M = \mathrm{Gal}(F^{(2)}/F) \simeq G/G''$, of the full tower group $G = \mathrm{Gal}(F^{(\infty)}/F)$ of the maximal unramified pro-p extension $F^{(\infty)} = \cup_{n\geq 1} F^{(n)}$ of F. According to the Artin reciprocity law, the latter can be computed numerically with the aid of kernels $\kappa(G) = (\ker(T_{F,E}))_E$ and targets $\tau(G) = (\mathrm{Cl}_p(E))_E$ of extension homomorphisms $T_{F,E} : \mathrm{Cl}_p(F) \to \mathrm{Cl}_p(E)$ of p-classes from F into all abelian unramified p-extensions $F \leq E \leq F^{(1)}$. The strategy has proved to be an outstanding innovation in computational class field theory and has been applied by myself and my international collaborators to base fields F with numerous types of p-class groups $\mathrm{Cl}_p(F)$ and primes $p \in \{2, 3, 5, 7\}$, starting with $(3, 3)$ in 2009 [1] and $(9, 3)$ in 2011, extended to three stages with Boston, Bush in 2012 [2], over $(2, 2, 2)$ with Azizi, Zekhnini, Taous in 2014 [3] and $(5, 5)$ with Azizi, Kishi, Talbi, Talbi in 2015 [4], up to the multi-layered situations $(4, 4)$ with Newman and $(9, 9)$, $(27, 3)$, $(81, 3)$ by myself in 2019, which led to my discovery of the surprising phenomenon of *harmonically balanced capitulation kernels*.
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Références

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